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Also solved by C. A. BARNHART, H. HALPERIN, H. L. OLSON, and A. PELLETIER.

2773 [1919, 212]. Proposed by JOSEPH ROSENBAUM, Milford, Conn.

Point out the fallacy in the proof of the following problem:

In the triangle $A_1B_1C_1$ let m be a point such that the sum of the distances from it to the sides is a maximum; also let $A_2B_2C_2$ be a triangle formed by drawing lines through the vertices A_1 , B_1 , and C_1 parallel to their opposite sides. Then the sum of the distances from m to the sides of the triangle $A_2B_2C_2$ is a minimum.

Proof.—Because the sides of the two triangles are parallel in pairs, the sum of the distances from a variable point P in triangle $A_1B_1C_1$ to the six sides of the two triangles is constant. Now by hypothesis M is a point for which one part of this constant sum is a maximum, and hence it follows that the other part is a minimum.

SOLUTION BY H. L. OLSON, University of Wisconsin.

This proof is correct, with the understanding that if a point P is on the opposite side of BC , for example, to the vertex A , the distance to the side BC is to be regarded as negative. It is easy to see, however, that the point M does not exist, and that the proposition is therefore vacuous. Represent the perpendicular distances from P to the sides BC , AC , and AB by α , β , and γ respectively. If we denote by Δ the area of the triangle ABC , we are to minimize the function $\alpha + \beta + \gamma$, subject to the condition $a\alpha + b\beta + c\gamma = 2\Delta$. (a , b , and c represent, as is customary, the sides BC , AC , and AB , respectively.) Eliminating γ , we have, as the function to be minimized,

$$\left(1 - \frac{a}{c}\right)\alpha + \left(1 - \frac{b}{c}\right)\beta + \frac{2\Delta}{c}.$$

Hence, the derivatives, $\left(1 - \frac{a}{c}\right)$, and $\left(1 - \frac{b}{c}\right)$, of this function with respect to α and β

must vanish; but for the general triangle they do not vanish and hence M does not exist. If, however, $a = b = c$, the sum of the distances is the constant $2\Delta/c$; likewise the sum of the distances for the corresponding triangle $A_2B_2C_2$ is constant.

Also solved by A. PELLETIER and A. L. WECHSLER.

2774 [1919, 212]. Proposed by FRANK IRWIN, University of California.

Evaluate the circulants

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ n & 1 & 2 & \cdots & n-2 & n-1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2 & 3 & 4 & \cdots & n & 1 \end{vmatrix}, \quad \text{and} \quad \begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-2} & a_{n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_2 & a_3 & a_4 & \cdots & a_n & a_1 \end{vmatrix},$$

where, in the latter, a_1, a_2, \dots, a_n form an arithmetical progression.

I. SOLUTION BY P. J. DA CUNHA, University of Lisbon, Portugal.

Denote the first of these circulants by Δ and the second by Δ^A . Let

$$s_n = \frac{1+n}{2}n$$

be the sum of the first n positive integers. Add to the elements of the last line of Δ the sum of the corresponding elements of all the preceding lines. We obtain a determinant which we can write as the product

$$\Delta = s_n \begin{vmatrix} 1 & 2 & 3 & \cdots & n-2 & n-1 & n \\ n & 1 & 2 & \cdots & n-3 & n-2 & n-1 \\ n-1 & n & 1 & \cdots & n-4 & n-3 & n-2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 4 & 5 & 6 & \cdots & 1 & 2 & 3 \\ 3 & 4 & 5 & \cdots & n & 1 & 2 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 1 \end{vmatrix}$$